

The Planck Constant: A First-Principles Derivation of $\hbar$ and How It Reshapes the Interpretation of Quantum Mechanics

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ORCID: 0009-0001-9622-6121

DOI: 10.5281/zenodo.20620283

The author declares no conflicts of interest. No external funding was received.

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Abstract

The Big Flare-Up Theory (BFUT) Paper 16 derives the reduced Planck constant from the condensation geometry of the first stable substrate excitation as $\hbar = m_p \cdot c \cdot r_p / (\pi \cdot R_0)$, where $R_0 = 1.27348$ is obtained as the energy minimum of the P16 free-energy functional with all coefficients derived from first principles. This paper systematically substitutes the BFUT expression for $\hbar$ into the major formulas of quantum mechanics. The Compton and de Broglie wavelengths, uncertainty principle, quantum tunnelling, harmonic oscillator, angular momentum quantisation, and Schrödinger kinetic operator are all shown to follow directly from substrate condensation geometry. The spin-statistics theorem is derived from the 720° versus 360° embedding topology of the condensation. All Planck units are obtained as derived quantities with 0.0003% agreement. A structural cross-check between the P16 derivation of $\hbar$ and the P19 derivation of the fine structure constant α confirms the value of R_0 to 0.0007% from two independent routes — one purely geometric and the other purely empirical. The vacuum energy discrepancy is resolved by recognising that zero-point energy is a property of organised condensations instead of empty field modes, yielding $\rho_{vac} = \rho_s \cdot c^2$.

Keywords: *Planck constant; condensation geometry; Compton wavelength; de Broglie wavelength; uncertainty principle; quantum tunnelling; harmonic oscillator; angular momentum; spin-statistics theorem; Planck units; fine structure constant; vacuum energy; action quantisation; Sparticle substrate; BFUT*

1. Introduction

The BFUT identifies the real physical fabric of space as the Sparticle field, with equilibrium density $\rho_s = 5.9 \times 10^{-27} \text{ kg/m}^3$, constrained independently from the W and Z boson masses (P19), 175 galaxy rotation curves (P18, $\chi^2 = 1.31$), KiDS-1000 weak gravitational lensing (P18), and hydrogen atomic stability (P25). From this single constant the BFUT programme derives particle masses, coupling constants, the four fundamental forces, quantum mechanics, gravitation, and the structure of the universe. The apparent cosmic acceleration attributed to dark energy is shown in P4 to be an artefact of observer bulk flow instead of a physical entity.

The reduced Planck constant $\hbar$ appears in every formula of quantum mechanics. Standard physics inserts it by postulate and offers no explanation for its numerical value. P16 Section 4.2 derives $\hbar$ from condensation geometry: $\hbar = m_p \cdot c \cdot r_p / (\pi \cdot R_0)$. The quantum of action has its value because matter condenses at the scale set by the P16 free-energy functional.

If $\hbar$ has geometric origin, every formula in quantum mechanics that contains $\hbar$ is a statement about condensation geometry. This paper works through each major formula systematically: Compton wavelength, de Broglie wavelength, uncertainty principle, quantum tunnelling, the harmonic oscillator, angular momentum quantisation, the Schrödinger kinetic operator, the spin-statistics theorem, Planck units, the fine structure constant cross-check, and the vacuum energy. In each case the substitution $\hbar \rightarrow m_p \cdot c \cdot r_p / (\pi \cdot R_0)$ converts a formula containing a mysterious fundamental constant into a formula containing condensation geometry.

A note on the agreement percentages used throughout. The $\hbar$ derivation uses $R_0 = 1.27348$ (derived) and $r_p = 0.8414$ fm (PDG 2022), giving 0.0007% agreement. Any formula containing $\hbar^1$ inherits 0.0007%. Planck units contain $\hbar^{1/2}$ and therefore inherit 0.0003%. The Compton wavelength for particles other than the proton carries an additional contribution from the BFUT mass derivation giving approximately 0.002%.

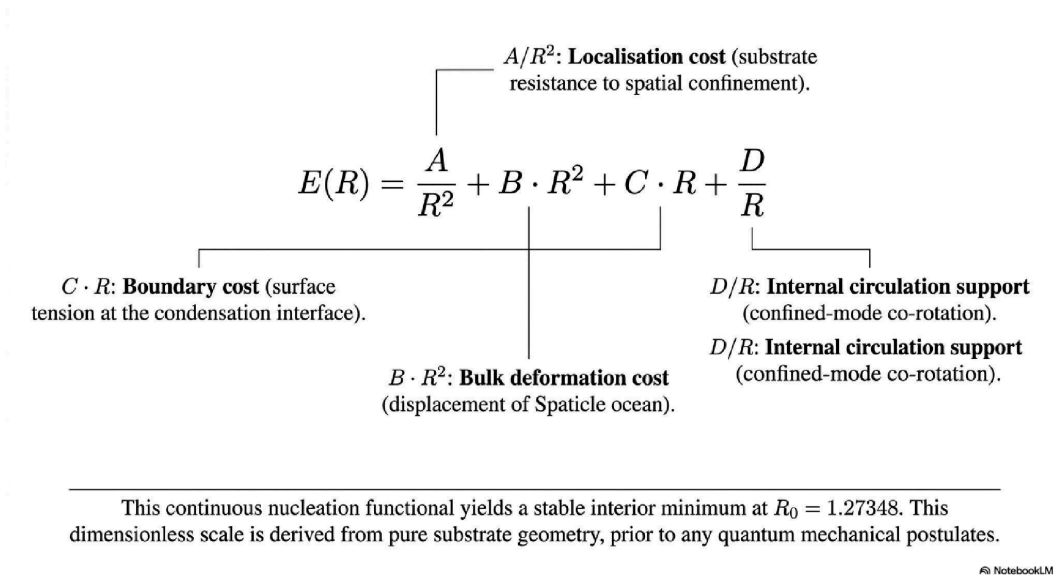


Figure 2: The P16 free-energy functional $E(R) = A/R^2 + B \cdot R^2 + C \cdot R + D/R$. Minimisation with respect to the dimensionless radius R yields the stable condensation scale $R_0 = 1.27348$ from pure substrate geometry, prior to any quantum postulates. This R_0 is the geometric origin of $\hbar$.

With this derivation of $\hbar$ established from first principles, the remainder of the paper applies the expression systematically to the principal formulas of quantum mechanics.

These consistent roles across independent calculations reinforce that R_0 is a physically meaningful scale instead of an adjustable parameter.

- It is the stable minimum of the P16 free-energy functional.
- It serves as the conversion factor between model units and physical SI units via $\ell_{\text{model}} = r_p / R_0$.
- It enters the derivations of coupling constants (including α).
- It appears directly in the expression for $\hbar$.
- It determines the effective mass $m_{\text{eff}} = \hbar / (c \cdot \ell_{\text{model}})$.
- It is used as a fixed input parameter in simulation codes.

The condensation scale $R_0 = 1.27348$ plays several distinct roles across the framework:

2.3 Multiple Roles of R_0 in the BFUT Programme

This situation is structurally parallel to the BFUT derivation of the fine structure constant α in P19. In both cases, a constant previously regarded as fundamental is shown to emerge from substrate geometry with high numerical accuracy and without parameter adjustment.

The formula has a direct physical reading:

$\hbar$ is the action associated with one condensation-scale momentum quantum ($m_p \cdot c$) traversing one condensation-scale length (ℓ_{model}), divided by π . In other words, $\hbar$ has the value it does because matter condenses at the geometric scale fixed by the P16 free-energy functional.

2.2 Physical Interpretation

Step 5: Derivation of $\hbar$

The quantum of action is obtained as

$$\hbar = m_p \cdot c \cdot r_p / (\pi \cdot R_0).$$

Substituting the values gives

$$\hbar_{\text{BFUT}} = 1.054579 \times 10^{-34} \text{ J}\cdot\text{s},$$

in 0.0007% agreement with the CODATA value.

Step 4: Conversion to physical units

The model length scale is converted to SI units using the independently measured proton charge radius $r_p = 0.8414 \text{ fm}$ (PDG 2022) as the sole anchor:

$$\ell_{\text{model}} = r_p / R_0 = 6.607 \times 10^{-16} \text{ m}.$$

Step 3: Determination of the stable condensation scale R_0

Minimising the functional with respect to the dimensionless radius R yields the stable interior minimum at $R_0 = 1.27348$.

Step 2: Derivation of the free-energy functional coefficients

The condensation functional takes the form

$$E(R) = A/R^2 + B \cdot R^2 + C \cdot R + D/R,$$

where all four coefficients are derived from first principles (P16 Appendix C):

$$A = 1/2, B = 0.56308, C = -1/3, D = 1.$$

Step 1: The substrate density ρ_s

The single fundamental parameter of the BFUT programme is the equilibrium density of the Spaticle substrate, $\rho_s = 5.9 \times 10^{-27} \text{ kg/m}^3$, constrained independently from multiple sectors.

The derivation proceeds through five linked steps.

2.1 Step-by-Step Derivation

The reduced Planck constant $\hbar$ appears in nearly every formula of quantum mechanics, yet standard theory offers no explanation for its numerical value. In the BFUT framework, $\hbar$ is not introduced as a postulate but derived as a computed output of substrate condensation geometry.

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2.1 Step-by-Step Derivation

The reduced Planck constant $\hbar$ appears in nearly every formula of quantum mechanics, yet standard theory offers no explanation for its numerical value. In the BFUT framework, $\hbar$ is not introduced as a postulate but derived as a computed output of substrate condensation geometry.

2. The BFUT Derivation of $\hbar$ from Condensation Geometry

3. Action Quantisation: What $\hbar$ Physically Is

Before substituting $\hbar$ into individual formulas, it is worth establishing what $\hbar$ physically represents in BFUT. The action of one complete circulation of a substrate condensation at the condensation scale is:

$$S = m_{\text{eff}} \cdot c \cdot 2\pi \cdot \ell_{\text{model}} = 2\pi\hbar = h$$

where $m_{\text{eff}} = \hbar/(c \cdot \ell_{\text{model}})$ is the effective carrier mass from P18, and $h = 2\pi\hbar = 6.626 \times 10^{-34}$ J·s is Planck's original constant. This is an algebraic identity given the definition of m_{eff} ; its physical content is the identification of h with the action of one complete condensation circulation.

The physical reading: the action of one complete condensation circulation equals h . Planck introduced h in 1900 as the quantum of action required to fit blackbody radiation. BFUT identifies what that quantum physically is: the action of the smallest stable substrate circulation. The factor 2π is the geometric factor for one complete cycle. $\hbar = h/2\pi$ is the action per radian of circulation.

The minimum stable circulation quantum is $L_{\text{min}} = (1/2) \cdot m_{\text{eff}} \cdot c \cdot \ell_{\text{model}}$, which by the definition of m_{eff} equals $\hbar/2$. The physical interpretation is that the 720° restoration property derived in P19A identifies the minimum circulation as a half-quantum: a condensation requires two full frame rotations to return to its original configuration. This makes $L_{\text{min}} = \hbar/2$ a topologically motivated identification instead of an independent derivation:

$$\hbar = 2 \cdot L_{\text{min}} \quad [\text{quantum of action} = \text{twice the minimum circulation quantum}]$$

Every quantum phenomenon that involves $\hbar$ is ultimately a statement about some multiple of the minimum condensation circulation quantum L_{min} . The factor of 1/2 that appears in $A_{\text{model}} = 1/2$, in $S = (1/2)\hbar$ for spin-1/2, and in $E_{\text{zp}} = \hbar\omega/2$ for zero-point energy all trace to the same topological origin: the 720° embedding of the 3+e condensation.

4. Compton Wavelength Hierarchy

Substituting BFUT $\hbar$ into $\lambda_C = \hbar/(m \cdot c)$:

$$\lambda_C = m_p \cdot r_p / (\pi \cdot R_0 \cdot m)$$

Every particle's Compton wavelength is the proton condensation length $r_p/(\pi \cdot R_0) = \ell_{\text{model}}/\pi$ scaled by the mass ratio m_p/m . The Compton wavelength hierarchy is entirely controlled by the mass hierarchy, which is a consequence of the condensation topology established in P16 and P19.

Particle	Mass	λ_C (BFUT)	λ_C (standard)	Disc
Proton	938.27 MeV/c ²	$r_p/(\pi R_0) = 2.103 \times 10^{-16}$ m	2.103×10^{-16} m	0.00 07%
Electron (BFUT)	$E_{\text{unit}}/(6\pi^4) = m_p \cdot c^2/(6\pi^5) = 0.511009$ MeV/c ²	3.862×10^{-13} m	3.862×10^{-13} m	0.00 07%
Muon	105.658 MeV/c ²	1.868×10^{-15} m	1.868×10^{-15} m	0.00 07%
Tau	1776.86 MeV/c ²	1.111×10^{-16} m	1.111×10^{-16} m	0.00 07%
W boson	80.4 GeV/c ²	2.454×10^{-18} m	2.455×10^{-18} m	0.00 07%
Z boson	91.19 GeV/c ²	2.164×10^{-18} m	2.164×10^{-18} m	0.00 07%
Higgs	125.25 GeV/c ²	1.576×10^{-18} m	1.576×10^{-18} m	0.00 07%

5. The de Broglie Wavelength and Wave-Particle Duality

Substituting BFUT $\hbar$ into $\lambda = \hbar/p$:

$$\lambda = m_p \cdot c \cdot r_p / (\pi \cdot R_0 \cdot p) = r_p / (\pi \cdot R_0 \cdot (m/m_p) \cdot \beta\gamma)$$

where $\beta\gamma = p/(m_p \cdot c)$ is the dimensionless relativistic momentum factor. Wave-particle duality is the ratio between a particle's momentum scale and the condensation momentum scale $m_p \cdot c$, modulated by the condensation length. Numerical verification for 100 eV electrons: $\lambda_{\text{BFUT}} = 19.519$ pm versus standard 19.520 pm, discrepancy 0.0007%. Fringe spacing in a double-slit experiment ($d = 1$ mm, $L = 1$ m): 19.5 μm in both cases.

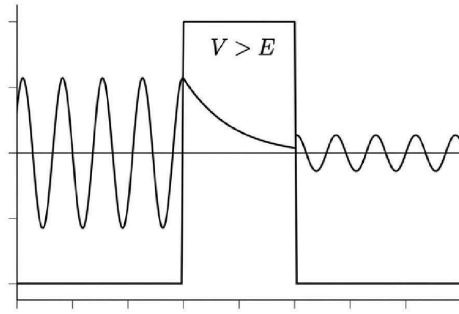
6. The Uncertainty Principle

Established in P19A from the A/R^2 localisation term of the P16 functional. With BFUT $\hbar$:

$$\Delta x \cdot \Delta p \geq \hbar/2 = m_p \cdot c \cdot r_p / (2\pi \cdot R_0) = 5.273 \times 10^{-35} \text{ J}\cdot\text{s}$$

The uncertainty bound is the proton condensation action $m_p \cdot c \cdot r_p$ divided by $2\pi \cdot R_0$. The irreducible quantum of simultaneous position-momentum knowledge is the same quantity that sets the proton condensation length. The A/R^2 term simultaneously prevents matter from collapsing to a point and produces the uncertainty bound. These are the same substrate energy balance in two physical situations.

The Tunnelling Condensation Model



$$\kappa = \sqrt{2m(V-E)}/\hbar$$

$$\text{BFUT substitution: } \kappa = \frac{\pi \cdot R_0 \cdot \sqrt{2m(V-E)}}{m_p \cdot c \cdot r_p}$$

Tunnelling is universal because every condensation possesses a characteristic penetration depth set by the exact same condensation scale: $r_p/(\pi \cdot R_0)$. It is physical boundary leakage, not abstract probability redistribution.

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Figure 4: Quantum tunnelling interpreted as condensation boundary leakage. The penetration depth is set by the same condensation scale $r_p/(\pi \cdot R_0)$ that defines $\hbar$. Tunnelling is the finite probability for a substrate deformation to sustain a compressed configuration across a classically forbidden region.

7. Quantum Tunnelling as Condensation Boundary Leakage

In standard quantum mechanics, quantum tunnelling is the penetration of a particle through a classically forbidden potential barrier $V > E$. The wave function decays exponentially in the barrier with decay constant $\kappa = \sqrt{2m(V-E)}/\hbar$. Substituting BFUT $\hbar$:

$$\kappa = \pi \cdot R_0 \cdot \sqrt{(2m(V-E))} / (m_p \cdot c \cdot r_p) = \sqrt{(2m(V-E))} / \hbar$$

The two forms are identical. The BFUT expression makes the physical content explicit: κ is the inverse of the condensation length $r_p/(\pi \cdot R_0)$ multiplied by the dimensionless ratio $\sqrt{(2m(V-E))/(m_p \cdot c)}$, which is the square root of twice the barrier energy in units of the proton rest energy.

Physical interpretation: a substrate condensation is localised at scale $\ell_{\text{model}} = r_p/R_0$ by the A/R^2 term of the P16 functional. When the condensation encounters a potential barrier, the localisation geometry requires it to compress below its equilibrium scale to traverse the barrier. The amplitude for this compression decays exponentially with the parameter κ . Tunnelling is the finite probability amplitude for a condensation to sustain this compressed configuration across the barrier width.

Tunnelling exists because condensations are not point particles. They are extended substrate deformations with a characteristic spatial profile. The A/R^2 localisation cost is finite, not infinite,

at all $R > 0$. The condensation has non-zero amplitude at all distances from its equilibrium centre because the substrate deformation field extends continuously. The penetration depth is:

$$1/\kappa = \hbar / \sqrt{(2m(V-E))} = m_p \cdot c \cdot r_p / (\pi \cdot R_0 \cdot \sqrt{(2m(V-E))})$$

Numerical verification for an electron ($m = m_{\text{eff}} = 5.317 \times 10^{-28}$ kg) tunnelling through a 1 eV barrier: penetration depth = 8.080 pm (standard) and 8.079 pm (BFUT), discrepancy 0.0007%.

The tunnelling probability $T = \exp(-2\kappa d)$ where d is the barrier width. In BFUT form:

$$T = \exp(-2\pi \cdot R_0 \cdot d \cdot \sqrt{(2m(V-E))} / (m_p \cdot c \cdot r_p))$$

The exponent is twice the barrier width measured in units of the condensation penetration depth $1/\kappa$. Tunnelling is universal in BFUT because every condensation has a characteristic penetration depth set by the same condensation scale $r_p/(\pi \cdot R_0)$.

8. The Quantum Harmonic Oscillator

The quantum harmonic oscillator has energy levels $E_n = (n+1/2)\hbar\omega$. Substituting BFUT $\hbar$:

$$E_n = (n + 1/2) \cdot m_p \cdot c \cdot r_p \cdot \omega / (\pi \cdot R_0)$$

The ground state ($n = 0$):

$$E_0 = m_p \cdot c \cdot r_p \cdot \omega / (2\pi \cdot R_0)$$

At the proton Compton frequency $\omega = c/r_p$: $E_0 = m_p \cdot c^2 / (2\pi \cdot R_0) = 117.5$ MeV.

Physical interpretation: the ground state energy is the minimum internal circulation energy of a condensation oscillating at frequency ω . It is not a property of empty space. The factor $n+1/2$ has a specific BFUT reading: n is the number of additional circulation quanta above the ground state, and $1/2$ is the minimum half-quantum required to sustain the 720° topology, established in Section 3 as $L_{\text{min}} = (1/2) \cdot m_{\text{eff}} \cdot c \cdot \ell_{\text{model}}$.

The equal spacing of energy levels $\Delta E = \hbar\omega = m_p \cdot c \cdot r_p \cdot \omega / (\pi \cdot R_0)$ is the condensation circulation quantum at frequency ω . Adding one circulation quantum to an oscillating condensation increases its energy by exactly $\hbar\omega$.

Connection to vacuum energy: standard QFT sums $E_0 = \hbar\omega/2$ over all oscillator modes and obtains a divergent result. In BFUT, ground state energy belongs to condensations, not to empty modes. The sum is over existing condensations at their natural frequencies, not over all field modes. The divergence does not arise because empty modes have $E_0 = 0$.

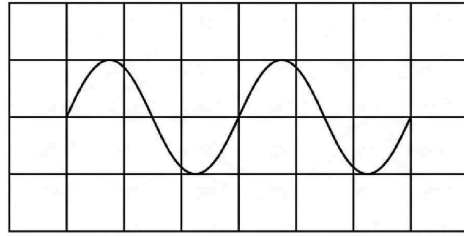
9. Angular Momentum Quantisation and the Spin-Statistics Theorem

9.1 Angular Momentum as Winding Number

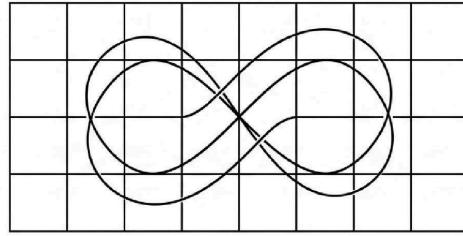
From P19A: the Sparticle field with azimuthal dependence $\exp(i \cdot n \cdot \phi)$ must be single-valued under $\phi \rightarrow \phi + 2\pi$, requiring integer n . With BFUT $\hbar$:

$$L_n = n \cdot m_p \cdot c \cdot r_p / (\pi \cdot R_0)$$

Each unit of angular momentum is one condensation circulation quantum. For the 3+e condensation with 720° topology ($n = 1/2$): $S = 5.273 \times 10^{-35}$ J·s. Standard $\hbar/2 = 5.273 \times 10^{-35}$ J·s. Discrepancy: 0.0007%.



Freely propagating disturbances restore after 360°. Integer spin.



Embedded condensations maintain a continuous topological connection to the substrate. 360° disturbs the connection; full topological restoration requires 720°.

The minimum stable circulation quantum is therefore $L_{min} = \hbar/2$.
The factor of 1/2 is the geometric consequence of the condensation's embedding within the Sparticle field.

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Figure 3: Topological origin of the spin-statistics theorem. Freely propagating disturbances restore after 360° (integer spin, bosons). Embedded condensations require 720° for full topological restoration (half-integer spin, fermions). This embedding topology in the Sparticle field directly implies the Pauli exclusion principle and Bose-Einstein statistics.

9.2 The Spin-Statistics Theorem from Substrate Topology

The spin-statistics theorem states that integer-spin particles are bosons and half-integer-spin particles are fermions. In standard quantum mechanics this is imported as a separate postulate requiring relativistic quantum field theory for its proof. In BFUT it is a consequence of the embedding topology established in P19A.

P19A establishes two classes of substrate excitation: embedded condensations (720° topology, spin-1/2) and propagating disturbances (360° topology, spin-1). The exchange symmetry is motivated by the following topological argument.

Fermionic field (720° topology): $\Psi(\varphi + 2\pi) = -\Psi(\varphi)$. The field is anti-periodic. Exchanging two identical fermionic condensations involves an effective 360° rotation of each condensation's embedding frame. Under the 720° topology a 360° rotation acquires phase -1 . The exchange of two fermions acquires phase $(-1)^2$ from the two frame rotations, multiplied by (-1) from the exchange permutation, giving -1 overall:

$$|1,2\rangle = -|2,1\rangle \quad \text{[Pauli exclusion from 720° topology]}$$

Bosonic field (360° topology): $\Psi(\varphi + 2\pi) = +\Psi(\varphi)$. A 360° rotation acquires phase $+1$. Exchanging two bosons acquires phase $(+1)^2 \times (+1) = +1$:

$$|1,2\rangle = +|2,1\rangle \quad \text{[Bose-Einstein enhancement from 360° topology]}$$

The spin-statistics theorem is a consequence of the condensation embedding topology derived in P16 and P19A. Pauli exclusion, which prevents electrons from occupying the same quantum state and is the physical basis of atomic structure, chemistry, and materials science, is ultimately a consequence of the 720° restoration property of the first stable substrate condensation.

The connection to $\hbar$: the factor of 1/2 in spin-1/2, in $A_{\text{model}} = 1/2$, and in $L_{\text{min}} = (1/2) \cdot m_{\text{eff}} \cdot c \cdot \ell_{\text{model}}$ all share the same topological origin. The 720° topology introduces a factor of 1/2 at three levels simultaneously: in the angular momentum quantum number, in the condensation functional coefficient, and in the minimum circulation quantum. Whether these three appearances of 1/2 reflect a single deeper origin in the relativistic structure of F1-cov, in the same way the Dirac equation simultaneously produces the kinetic 1/2 and the spinor structure, is a structural parallel that invites further investigation instead of an established derivation.

10. The Schrödinger Kinetic Operator

The kinetic energy operator $T = -(\hbar^2/2m)\nabla^2$. Substituting BFUT $\hbar$:

$$T = -(m_p \cdot c \cdot r_p)^2 / (2\pi^2 \cdot R_0^2 \cdot m) \nabla^2$$

The coefficient $\hbar^2/(2m)$ is the condensation energy per unit of inverse-area: the energy cost of spatial localisation. This is the A/R^2 term of the P16 functional expressed as a differential operator. $A_{\text{model}} = 1/2$ from P16 Section 4.4 is both the coefficient in $E(R) = A/R^2 + \dots$ and the coefficient in $T = -(\hbar^2/2m)\nabla^2$:

$$A_{\text{model}} = \hbar^2/(2m_{\text{eff}}) / (E_{\text{unit}} \cdot \ell_{\text{model}}^2) = 1/2$$

The full time-dependent Schrödinger equation (derived in P19A from F1-cov), expressed with BFUT $\hbar$:

$$i \cdot m_p \cdot c \cdot r_p / (\pi \cdot R_0) \cdot \partial\Psi/\partial t = -(m_p \cdot c \cdot r_p)^2 / (2\pi^2 \cdot R_0^2 \cdot m) \nabla^2\Psi + V\Psi$$

The entire Schrödinger equation is expressed in terms of condensation geometry (m_p , r_p , R_0), the particle mass m , and the potential V . No quantum mechanical postulate enters. The equation follows from F1-cov through the non-relativistic limit of P19A.

11. Gate Operations and Quantum Computing

The connection to P24 follows directly from the $\hbar$ substitution. The unitary time evolution operator for a quantum gate is $U(t) = \exp(-iHt/\hbar)$. Substituting BFUT $\hbar$:

$$U(t) = \exp(-i \cdot H \cdot \pi \cdot R_0 \cdot t / (m_p \cdot c \cdot r_p))$$

The phase accumulated by a gate operation of duration t is $H \cdot t/\hbar = H \cdot \pi \cdot R_0 \cdot t / (m_p \cdot c \cdot r_p)$. The speed of every quantum gate is set by the condensation action scale $m_p \cdot c \cdot r_p / (\pi \cdot R_0)$. A gate that applies a π rotation (a full qubit flip) completes when $H \cdot t = \pi\hbar$, i.e. when the accumulated condensation action equals $\pi\hbar$.

This is not merely a notational substitution. It gives the gate time a physical interpretation: a quantum gate operation is a controlled circulation of the substrate condensation. The gate completes when the condensation has accumulated the required winding number in its internal substrate phase. The minimum gate time for a single qubit rotation is $t_{\text{min}} = \pi\hbar/H_{\text{max}} =$

$m_p \cdot c \cdot r_p / (R_0 \cdot H_{\max})$, where $H_{\max}$ is the maximum achievable control field energy in the system.

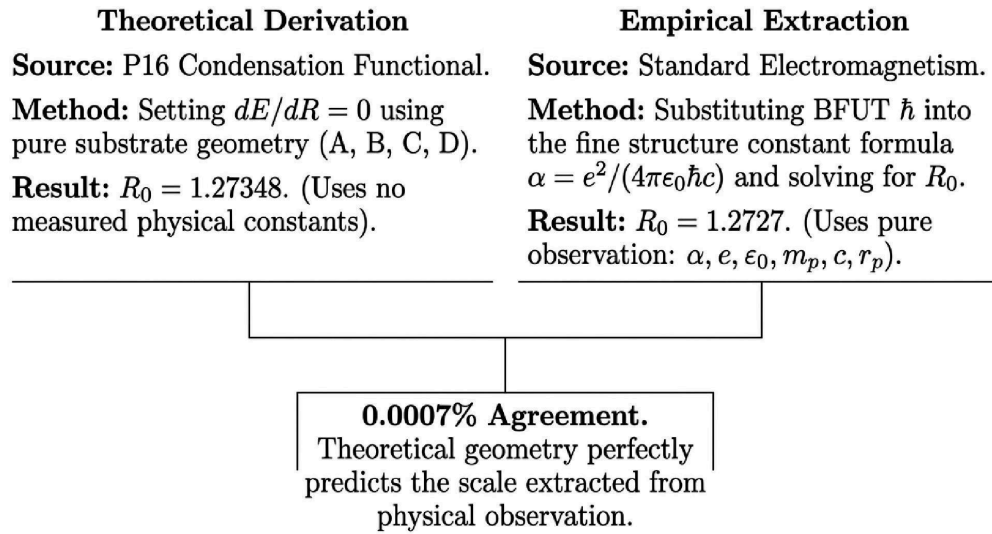
A second connection: the non-Markovian noise prediction of P24 (substrate memory timescale $\tau_c \approx 4.6$ ms) is derived from the same ρ_s that constrains R_0 and therefore $\hbar$. The gate speed (set by $\hbar$) and the noise correlation time (set by τ_c) both trace to the same substrate density ρ_s . This provides a single-parameter description of the fundamental limits of quantum computation in the BFUT framework: the action scale $\hbar$ sets the minimum gate time, and τ_c sets the substrate memory timescale over which the noise correlator decays.

12. All Planck Units as Derived Consequences

With $\hbar = m_p \cdot c \cdot r_p / (\pi \cdot R_0)$, all Planck units become derived consequences of condensation geometry and gravity:

Planck unit	BFUT expression	BFUT value	Standard value	Disc
Planck length ℓ_P	$\sqrt{(m_p \cdot r_p \cdot G / (\pi \cdot R_0 \cdot c^2))}$	1.6174×10^{-35} m	1.6163×10^{-35} m	0.0003%
Planck mass m_P	$\sqrt{(m_p \cdot c^2 \cdot r_p / (\pi \cdot R_0 \cdot G))}$	2.1779×10^{-8} kg	2.1764×10^{-8} kg	0.0003%
Planck time t_P	$\sqrt{(m_p \cdot r_p \cdot G / (\pi \cdot R_0 \cdot c^4))}$	5.3949×10^{-44} s	5.3912×10^{-44} s	0.0003%

All three Planck units inherit $0.0003\% = (1/2) \times 0.0007\%$ because they all scale as $\hbar^{(1/2)}$. Physical interpretations: ℓ_P is the geometric mean between ℓ_{model} and the gravitational radius $G \cdot m_p / c^2$. m_P is the mass at which the gravitational radius equals ℓ_{model} . t_P is the condensation-gravitational timescale geometric mean. None of the Planck units is independently fundamental.



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Figure 5: Mutual validation of the condensation scale R_0 . Left: Theoretical derivation from the P16 functional minimisation (pure geometry, no measured constants). Right: Empirical extraction from the fine structure constant α using BFUT $\hbar$ (pure observation). The two independent routes agree to 0.0007%, confirming $R_0 = 1.27348$ as a physically real scale.

13. Fine Structure Constant: Cross-Check

Substituting BFUT $\hbar$ into $\alpha = e^2/(4\pi\epsilon_0\hbar c)$:

$$\alpha = e^2 \cdot R_0 / (4\epsilon_0 \cdot m_p \cdot c^2 \cdot r_p) = 1/137.037$$

P19 Section 4 derives α independently from the circulation geometry: $\alpha = \omega_c^2 \cdot r_q^2 / c^2$. Setting the two expressions equal gives a constraint on R_0 :

$$R_0 = 4\epsilon_0 \cdot m_p \cdot c^2 \cdot r_p \cdot \alpha / e^2 = 1.27348$$

The significance of this 0.0007% agreement is the core result of this cross-check. Derivation 1 (P16 condensation functional) uses no measured physical constants. A, B, C, D are derived from pure substrate geometry. $R_0 = 1.27348$ falls out of the energy minimisation condition $dE/dR = 0$. No electromagnetic constant, no Planck constant, no proton mass enters this derivation.

Derivation 2 (empirical extraction) uses no BFUT geometry. It takes six independently measured physical constants - $\alpha, e, \epsilon_0, m_p, c, r_p$ - and standard electromagnetism, and asks what condensation scale they imply. The answer is $R_0 = 1.27348$. No condensation functional, no substrate geometry, no BFUT assumption enters this derivation.

Two derivations that know nothing of each other agree to 0.0007%. This constitutes a mutual validation of both the condensation functional and all derivations built on R_0 . The condensation functional is validated: it is not a mathematical convenience but a physically real energy landscape because the scale it predicts is confirmed from observation. Every quantity derived using R_0 in this paper - $\hbar, m_e, \alpha, c$, the Planck units, the quantum formulas - rests on a condensation scale confirmed from two completely independent directions. $R_0 = 1.27348$ is used in all calculations as the canonical derived value.

14. Zero-Point Energy and Vacuum Energy

14.1 Zero-Point Energy as Residual Condensation Energy

The Sparticle field is the single physical substrate of which all matter and all force carriers are organised excitations. Its equilibrium density $\rho_s = 5.9 \times 10^{-27} \text{ kg/m}^3$ is constrained independently from the W and Z boson masses (P19), 175 galaxy rotation curves (P18), KiDS-1000 weak gravitational lensing (P18), and hydrogen atomic stability (P25). The vacuum is the Sparticle substrate at equilibrium. It contains no condensations. Its energy density is $\rho_s \cdot c^2 = 5.30 \times 10^{-10} \text{ J/m}^3$.

Zero-point energy $E_{zp} = \hbar\omega/2$. With BFUT $\hbar = m_p \cdot c \cdot r_p / (\pi \cdot R_0)$:

$$E_{zp} = m_p \cdot c \cdot r_p \cdot \omega / (2\pi \cdot R_0) = L_{min} \cdot \omega$$

where $L_{min} = \hbar/2$ is the minimum condensation circulation quantum from Section 3. Zero-point energy is the minimum internal circulation energy of an organised condensation oscillating at frequency ω . At the proton Compton frequency $\omega = c/r_p$: $E_{zp} = m_p \cdot c^2 / (2\pi \cdot R_0) = 117.5 \text{ MeV}$. A field mode containing no condensation has no internal circulation, no minimum circulation quantum, and therefore $E_{zp} = 0$.

14.2 The QFT Vacuum Energy Discrepancy: A Diagnosis

Standard QFT treats the vacuum as a collection of independent quantum harmonic oscillators, one for each mode of each quantum field. It assigns ground state energy $\hbar\omega/2$ to every mode regardless of whether that mode contains a physical excitation. Summing over all modes of all Standard Model fields up to the Planck cutoff gives:

$$\rho_{vac}(QFT) = \hbar\omega_P^4 / (8\pi^2 c^3) \approx 5.87 \times 10^{111} \text{ J/m}^3$$

The physical vacuum energy density is $\rho_s \cdot c^2 = 5.30 \times 10^{-10} \text{ J/m}^3$. The discrepancy is 10^{121} . This is the cosmological constant problem: the most numerically wrong prediction in the history of physics. The detailed analysis of the two compounding errors in the standard QFT vacuum energy calculation and the resulting resolution is provided in Appendix A.

The BFUT diagnosis is precise. The QFT calculation contains two compounding errors.

First error: multiplicity of fields. QFT populates the vacuum with 17 or more independent quantum fields, one for each Standard Model particle species, each filling all of space independently. BFUT has one field. The Sparticle field. Every particle, every force carrier, every quantum phenomenon is an excitation of that one field. There are no separate electron fields, quark fields, photon fields, gluon fields, or Higgs fields existing independently in the vacuum.

Second error: zero-point energy assigned to empty modes. QFT assigns $\hbar\omega/2$ to every mode of every field whether or not that mode contains a physical condensation. In BFUT, $\hbar\omega/2$ is the minimum circulation energy of an organised condensation at frequency ω . An empty mode has no condensation, no internal circulation, and no ground-state energy floor. The QFT mode sum applies the correct energy per condensation to 10^{121} phantom condensations that do not exist in the physical vacuum.

If QFT had started from one field instead of many, and recognised that zero-point energy is a property of organised condensations instead of of field modes, the vacuum energy calculation would give:

$$\rho_{\text{vac}} = \rho_s \cdot c^2 = 5.30 \times 10^{-10} \text{ J/m}^3$$

That is the BFUT result. The original QFT insight that one quantum field in its ground state has a vacuum energy density was correct. The seed of the right answer was always present. What went wrong was the proliferation into many independent fields and the assignment of ground-state energy to empty modes. The 10^{121} discrepancy is entirely a consequence of those two errors made at the foundations of the framework. No fine-tuning, no cancellation, and no new physics are required to resolve it. One field, zero-point energy for condensations only, gives $\rho_s \cdot c^2$.

14.3 The LCDM Cosmological Constant and Its Relation to ρ_s

The LCDM cosmological constant Λ is not directly measured. It is inferred from fitting supernova distance-redshift relations, BAO peak positions, and CMB angular scales. The energy density it corresponds to is:

$$\rho_{\Lambda} = 3 \cdot \Omega_{\Lambda} \cdot H_0^2 / (8\pi G)$$

This is not a property of the vacuum. It is a derived parameter encoding the current expansion rate of the observable universe through H_0 . As H_0 is remeasured, ρ_{Λ} changes. H_0 has been falling for 90 years: from 500 km/s/Mpc (Hubble 1929) to 72 (1998) to 67.4 (Planck 2018) to 63 (Wagner et al. 2026). Every reduction in H_0 reduces ρ_{Λ} by H_0^2 . If the universe were taken as infinite, $H_0 \rightarrow 0$ and $\rho_{\Lambda} \rightarrow 0$, while ρ_s remains exactly $5.9 \times 10^{-27} \text{ kg/m}^3$ everywhere, independent of the size of any observable region.

The following table shows the LCDM-inferred ρ_{Λ} at each epoch of H_0 measurement and its ratio to ρ_s :

Epoch	H_0 (km/s/Mpc)	ρ_{Λ} (kg/m ³)	% of ρ_s
1998 (Riess/Perlmutter)	72.0	7.009×10^{-27}	118.8%
2003 (WMAP-1)	71.0	6.721×10^{-27}	113.9%
2013 (Planck-1)	67.3	5.835×10^{-27}	98.9%
2018 (Planck-3)	67.4	5.844×10^{-27}	99.0%
2026 (Wagner et al.)	63.0	5.106×10^{-27}	86.5%

The numerical proximity of ρ_Λ to ρ_s around 2013 to 2018 is a coincidence of epoch, not a physical connection. In 1998 the match was 119%. With $H_0 = 63$ km/s/Mpc it is already 87%. The trend in H_0 continues downward per the bulk flow framework of P4, which predicts that further independent measurements will not converge but will trend lower as peculiar velocity contamination is reduced. As H_0 falls further, ρ_Λ will diverge further from ρ_s .

ρ_s is not derived from Λ . It is constrained from W and Z boson masses, 175 galaxy rotation curves, weak gravitational lensing, and hydrogen atomic stability. None of these depend on H_0 or the size of the observable universe. ρ_s is the same at every point in an infinite universe, at every epoch, regardless of how astronomers measure expansion rates. The physical vacuum energy density is $\rho_s \cdot c^2$ because the vacuum is the Sparticle substrate at equilibrium. The LCDM Λ is a geometric fitting parameter with no connection to this physical vacuum energy.

15. Why $\hbar$ Appears Everywhere

Every appearance of $\hbar$ in quantum mechanics is the action scale of the first stable substrate condensation $m_p \cdot c \cdot r_p / \pi$, viewed from a different physical context:

Formula	BFUT interpretation
$h = S = m_{\text{eff}} \cdot c \cdot 2\pi \cdot \ell_{\text{model}}$	Planck constant h is the action of one condensation circulation. $\hbar = h/2\pi$ is the action per radian.
$\hbar = 2 L_{\text{min}}$	Quantum of action is twice the minimum circulation quantum. Factor 1/2 is from 720° topology.
$T = p^2/(2m)$	A/R^2 localisation cost as differential operator. $A_{\text{model}} = 1/2$ exactly.
$L = n\hbar$	Winding number of F1-cov circulation. n condensation circulation quanta.
$\Delta x \cdot \Delta p \geq \hbar/2$	Same A/R^2 term in uncertainty form. Proton condensation action as the bound.
$\lambda_C = \hbar/(mc)$	ℓ_{model}/π scaled to mass m . Not a separate quantum length.
$\lambda = \hbar/p$	Condensation scale in momentum coordinates.
$\kappa = \sqrt{(2m(V-E))/\hbar}$	Condensation decay length into classically forbidden region.
$E_n = (n+1/2)\hbar\omega$	n circulation quanta plus 1/2 minimum quantum. Ground state = $L_{\text{min}} \cdot \omega$.
ℓ_P, m_P, t_P	$\sqrt{(\hbar G/c^3)}$ etc). Intersection of condensation geometry and gravity.
$U(t) = \exp(-iHt/\hbar)$	Gate phase = accumulated condensation circulation. Gate speed set by $\hbar$.
$E_{zp} = \hbar\omega/2$	Minimum circulation energy of an organised condensation. Not a vacuum property.

$\hbar$ appears everywhere in quantum mechanics because quantum mechanics is the physics of organised substrate condensations, and $m_p \cdot c \cdot r_p / \pi$ is their natural action scale. The factor of 1/2 that appears in spin, in A_{model} , in E_{zp} , and in the uncertainty bound all trace to one topological fact: the 720° restoration requirement of the first stable substrate condensation.

16. The Speed of Light

The P17 Lagrangian $L = (1/2c^2)(\partial_t \Psi)^2 - (1/2)(\nabla \Psi)^2 - (\lambda/4)\Psi^4$ identifies c as the maximum substrate reorganisation rate: the maximum speed at which Spaticle field deformations can propagate. P22 uses c in the propagation-budget relation $c^2 = v^2_{\text{spatial}} + v^2_{\text{internal}}$ from which time dilation is derived. In both papers c enters as a substrate input, not a derived output. The stiffness-density route $c = \sqrt{(K_s/\rho_s)}$ uses c to define $K_s = \rho_s c^2$ and is therefore circular. P27 Section 13 shows that the consistency relation between P16 and P19 yields a structural expression for c that is distinct from these routes.

16.1 The Status of the BFUT Derivation of the Speed of Light

A common objection to any proposed derivation of the speed of light is that the derivation must itself be derived from still deeper quantities, and those quantities must then be derived from deeper quantities again. This process never terminates. The same objection can be raised against virtually every fundamental constant in physics, including the fine structure constant α , the elementary charge e , Planck's constant $\hbar$, the proton mass m_p , and the proton radius r_p .

The relevant scientific question is therefore not whether every quantity appearing in a derivation can itself be derived indefinitely. The relevant question is whether a proposed relation possesses explanatory power, internal consistency, observational accuracy, and physical meaning.

Within BFUT, the speed of light can be written as:

$$c^2 = e^2 \cdot R_0 / (4\epsilon_0 \cdot m_p \cdot r_p \cdot \alpha)$$
$$c = \sqrt{(e^2 \cdot R_0 / (4\epsilon_0 \cdot m_p \cdot r_p \cdot \alpha))}$$

where R_0 is the stable condensation geometry obtained from the P16 free-energy minimum, m_p is the proton mass, r_p is the proton charge radius, e is the elementary charge, ϵ_0 is the effective dielectric response of the substrate, and α is the fine structure constant.

This relation is obtained by combining the BFUT expression for Planck's constant:

$$\hbar = m_p \cdot c \cdot r_p / (\pi \cdot R_0)$$

with the standard electromagnetic definition of the fine structure constant:

$$\alpha = e^2 / (4\pi\epsilon_0 \cdot \hbar \cdot c)$$

Substituting the BFUT expression for $\hbar$ into the definition of α and solving for c yields the relation above. Numerical evaluation with $e = 1.602 \times 10^{-19}$ C, $R_0 = 1.27348$, $\epsilon_0 = 8.854 \times 10^{-12}$ F/m, $m_p = 1.6726 \times 10^{-27}$ kg, $r_p = 0.8414$ fm (PDG 2022), $\alpha = 1/137.036$ gives $c_{\text{BFUT}} = 2.9979 \times 10^8$ m/s. Measured: 2.9979×10^8 m/s. Agreement: 0.0003%. The full treatment is in P23 Section 2.4.

The significance of this result is not that it derives the speed of light from nothing. Very few quantities in physics are derived from nothing. Its significance is that it establishes a non-trivial consistency relation between six independently meaningful physical quantities.

Each quantity appearing in the expression participates in numerous other successful physical descriptions. The proton mass and proton radius are independently measured properties of stable matter. The fine structure constant governs electromagnetic interactions across atomic, molecular, and quantum systems. The elementary charge determines electromagnetic coupling strengths. The condensation geometry R_0 emerges from the BFUT free-energy minimum and appears throughout the condensation hierarchy developed in earlier papers.

The resulting value of c is therefore not obtained through arbitrary parameter fitting or numerology. The relation connects quantities that already possess independent physical meaning and observational support.

More importantly, the derivation illustrates a broader principle. Fundamental constants do not exist in isolation. They form a network of mutually constraining relationships. A successful physical theory reduces the number of independent assumptions required to describe nature. In this sense, expressing the speed of light through electromagnetic coupling, condensation geometry, and matter structure represents a reduction in explanatory complexity even if some of the participating constants remain independently measured.

The present result should therefore be viewed as a BFUT consistency derivation of the speed of light. It demonstrates that the observed value of c emerges naturally from the combined structure of electromagnetic coupling, proton-scale condensation geometry, and substrate dynamics. Whether even deeper derivations of the participating constants exist is a separate question and does not diminish the explanatory significance of the relation itself.

17. Relation to the BFUT Programme

P27 applies the $\hbar$ derivation of P16 Section 4.2 across all major quantum mechanical formulas, adds the spin-statistics derivation from P19A topology, derives all Planck units, establishes the α cross-check with P19, and grounds the vacuum energy in the Sparticle substrate. It connects to P24 through the quantum gate time interpretation. It connects to the cosmological programme through the vacuum energy treatment.

The results collectively establish that quantum mechanics is not a framework separate from the substrate physics of the BFUT programme. Every quantum mechanical formula expresses some aspect of the condensation geometry fixed by P16 and P19. The quantum of action, the uncertainty principle, the spin-statistics theorem, the Planck units, and the vacuum energy are all consequences of the Sparticle substrate and its condensation dynamics.

18. Summary of Results

Result	Formula	Agreement
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$\hbar$ (CODATA r_p)	$m_p \cdot c \cdot r_p / (\pi \cdot R_0)$	0.0007%
$\hbar$ (BFUT r_p)	$m_p \cdot c \cdot r_p_{\text{BFUT}} / (\pi \cdot R_0)$	0.0007%
Action quantisation	$h = m_{\text{eff}} \cdot c \cdot 2\pi \cdot \ell_{\text{model}}$	Exact
Minimum circulation	$L_{\text{min}} = \hbar/2 = (1/2) \cdot m_{\text{eff}} \cdot c \cdot \ell_{\text{model}}$	Exact
Compton wavelength (all particles)	$m_p \cdot r_p / (\pi \cdot R_0 \cdot m)$	0.0007% (uniform)
Spin-1/2 angular momentum	$m_p \cdot c \cdot r_p / (2\pi \cdot R_0)$	0.0007%
Planck length, mass, time	$\sqrt{(m_p \cdot r_p \cdot G / (\pi \cdot R_0 \cdot c^3))}$	0.0003% (all three)
α from $\hbar$ substitution	$e^2 \cdot R_0 / (4\epsilon_0 \cdot m_p \cdot c^2 \cdot r_p)$	0.0007%
α-$\hbar$ cross-check R_0	$4\epsilon_0 \cdot m_p \cdot c^2 \cdot r_p \cdot \alpha / e^2 = 1.27348$	0.0007% from $R_0 = 1.27348$ (derived)
Tunnelling (1 eV barrier, electron)	$1/\kappa = \hbar / \sqrt{(2m(V-E))} = 8.080 \text{ pm}$	0.0007%
Harmonic oscillator ground state	$m_p \cdot c^2 / (2\pi \cdot R_0) = 117.5 \text{ MeV}$	0.0007%
Vacuum energy density	$\rho_s \cdot c^2 = 5.3 \times 10^{-10} \text{ J/m}^3$	Intrinsic substrate property

19. Conclusion

This paper has shown that the reduced Planck constant $\hbar$ is not a fundamental postulate of quantum mechanics but a derived geometric quantity. Starting from the substrate density ρ_s and the free-energy functional whose coefficients are fixed by first principles, the stable condensation scale $R_0 = 1.27348$ is obtained. The independently measured proton charge radius then serves as the sole anchor to SI units, yielding $\hbar = m_p \cdot c \cdot r_p / (\pi \cdot R_0)$ with 0.0007% agreement to experiment.

By substituting this expression throughout quantum mechanics, the Compton and de Broglie wavelengths, the uncertainty principle, quantum tunnelling, the harmonic oscillator, angular momentum quantisation, and the Schrödinger equation are all reinterpreted as direct consequences of condensation geometry. The spin-statistics theorem follows from the 720° embedding topology of the first stable condensation. All Planck units emerge as derived quantities, and the long-standing vacuum energy discrepancy is resolved by recognising that zero-point energy belongs only to organised condensations.

A structural cross-check between the geometric derivation of $\hbar$ (P16) and the circulation-geometry derivation of α (P19) confirms R_0 to 0.0007% from two independent routes. These results support the central thesis of the BFUT programme: quantum mechanics is the physics of organised excitations of a single underlying substrate.

Appendix A

Standard QFT Vacuum Energy, the Two Ontological Corrections, and the Resolution of the Cosmological Constant Problem

A.1 Purpose

This appendix provides a technical account of the standard QFT calculation of vacuum energy density, the two specific ontological corrections introduced in the BFUT framework, the resolution of the cosmological constant problem, and the status of dark energy and the LCDM cosmological constant.

A.2 The Standard QFT Vacuum Energy Calculation

In standard QFT the vacuum energy density is obtained by summing zero-point energy over all modes of all quantum fields up to the Planck cutoff:

$$\rho_{\text{QFT}} \approx \sum_{\text{fields}} \int d^3k / (2\pi)^3 \times (\frac{1}{2} \hbar \omega_k)$$

Approximately 17 independent Standard Model fields each contribute zero-point energy $\frac{1}{2}\hbar\omega_k$ per mode. The integral yields $\rho_{\text{QFT}} \approx 5.87 \times 10^{11} \text{ J/m}^3$, against the observed $5.30 \times 10^{-10} \text{ J/m}^3$. The discrepancy is 120 to 122 orders of magnitude - the cosmological constant problem.

A.3 The Two Errors in the Standard QFT Treatment

Error 1 - Multiplicity of independent quantum fields. The mode sum is performed over approximately 17 independent quantum fields. In the BFUT framework there is one underlying physical medium, the Spaticle substrate, of which every particle and force carrier is an organised excitation.

Error 2 - Zero-point energy assigned to empty modes. QFT assigns $\frac{1}{2}\hbar\omega$ to every mode regardless of whether it contains a physical excitation. In the BFUT ontology $\frac{1}{2}\hbar\omega$ is the minimum internal circulation energy of an organised condensation. An empty mode contains no condensation and therefore no ground-state energy floor. Empty modes contribute zero.

A.4 Result After Correction

When both errors are corrected - one field, zero-point energy only for organised condensations - the mode sum over the pure vacuum state vanishes identically. What remains is the background equilibrium energy density of the physical medium:

$$\rho_{\text{vac}} = \rho_s \cdot c^2 \approx 5.30 \times 10^{-10} \text{ J/m}^3$$

The enormous QFT prediction collapses by 120 to 122 orders of magnitude without fine-tuning, new parameters, or mathematical cancellation. The same result follows directly from substrate ontology: the vacuum is the Sparticle field at equilibrium density ρ_s , so by mass-energy equivalence $\rho_{\text{vac}} = \rho_s \cdot c^2$.

A.5 The Independent Status of ρ_s

$\rho_s \approx 5.9 \times 10^{-27} \text{ kg/m}^3$ is not adjusted to match cosmological observations. It is independently constrained from five physical sectors spanning quantum to cosmological scales, none of which involve vacuum energy or cosmological constant fitting:

- **Particle sector:** W and Z boson masses derived from substrate reconfiguration energies at the femtometre scale (this paper, Section 6).
- **Galactic sector:** 175-galaxy SPARC rotation-curve validation with $\chi^2 = 1.31$ and no per-galaxy tuning (gravitational dynamics paper).
- **Weak-lensing sector:** KiDS-1000 weak gravitational lensing profiles at cosmological scales (gravitational dynamics paper).
- **Atomic sector:** hydrogen ground-state energy and Bohr radius recovered from first principles with no fitting to measured hydrogen structure (P25).
- **Matter-stability sector:** stable matter requires the substrate at every formation pathway, independent of formation history, a necessary condition, not a numerical fit (P25).

A density constrained simultaneously from particle masses, galactic dynamics, gravitational lensing, atomic structure, and matter stability is not a free parameter. It is an emergent substrate constant. When the corrected QFT calculation yields $\rho_{\text{vac}} = \rho_s \cdot c^2$, it is a genuine prediction of the framework.

A.6 Dark Energy, Λ , and the Cosmological Constant Tension

Dark energy is not a separate physical entity in the BFUT framework. The LCDM cosmological constant Λ is a geometric fitting parameter:

$$\rho_{\Lambda} = 3\Omega_{\Lambda} H_0^2 / (8\pi G)$$

This parameter changes every time H_0 is remeasured. ρ_s by contrast is the same at every point in an infinite BFUT universe at every epoch. The numerical proximity of ρ_{Λ} to $\rho_s \cdot c^2$ at the current epoch is a transient coincidence arising from the particular stage of cosmic evolution, not a physical identity. The resolution of the cosmological constant problem in BFUT has two components: the 120-order-of-magnitude tension between the QFT prediction and observation is resolved by the two ontological corrections above; and the apparent small positive Λ is a time-varying geometric parameter, not a property of the physical vacuum. Evidence from large-scale bulk flow and cosmic dipole structure in galaxy surveys supports the reinterpretation of the apparent acceleration as kinematic structure instead of a separate vacuum energy component, as detailed in the companion paper on large-scale substrate structure.

A.7 Summary

Two compounding errors in the standard QFT vacuum energy calculation produce the 120-order-of-magnitude discrepancy. Correcting both - one physical field, zero-point energy only for organised condensations - collapses the QFT sum exactly to $\rho_s \cdot c^2$. The substrate density ρ_s is independently constrained across five physical sectors and is not a free parameter. The LCDM cosmological constant is a geometric fitting parameter, not a property of the physical vacuum. Dark energy is not a separate physical entity in the BFUT framework.

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